JMO Miscellaneous Questions

Level: Junior Ref No: J12

Puzz Points: 15

[JMO 2000 B6] X and Y play a game in which X starts by choosing a number, which must be either 1 or 2.

Y then adds either 1 or 2 and states the total of the two numbers chosen so far. X does likewise, adding either 1 or 2 and stating the total, and so on. The winner is first player to make the total reach (or exceed) 20.

- (i) Explain how X can always win.
- (ii) The game is now modified so that at each stage the number chosen must be 1 or 2 or 4.Which of X or Y can now always win and how?

Solution: (i) X chooses 2 first. Whatever Y chooses, X chooses the other. (ii) 'Winning positions' are 3, 6, 9, 12, 15: a player achieving any of these totals can win by 'best play'. Player with total 1 or 2 or 4 or 5 or 7 etc. gives opponent opportunity of establishing a winning position.