## JMO Miscellaneous Questions

Level: Junior Ref No: J12
Puzz Points: 15
[JMO 2000 B6] X and Y play a game in which X starts by choosing a number, which must be either 1 or 2.
$Y$ then adds either 1 or 2 and states the total of the two numbers chosen so far. $X$ does likewise, adding either 1 or 2 and stating the total, and so on. The winner is first player to make the total reach (or exceed) 20.
(i) Explain how $X$ can always win.
(ii) The game is now modified so that at each stage the number chosen must be 1 or 2 or 4 . Which of $X$ or $Y$ can now always win and how?

Solution: (i) $X$ chooses 2 first. Whatever $Y$ chooses, $X$ chooses the other. (ii) 'Winning positions' are 3, $6,9,12,15$ : a player achieving any of these totals can win by 'best play'. Player with total 1 or 2 or 4 or 5 or 7 etc. gives opponent opportunity of establishing a winning position.

